Kirchhoff' Law

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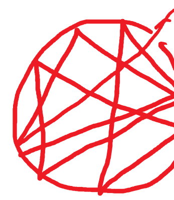
(i) absorption (ii) Emission

A → Smooth B → Rough

∴ Surface A is a poor absorber Compared to the surface B

(i) Poor absorber (ii) Good absorber

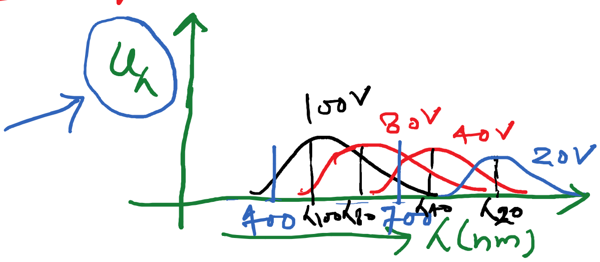
(iii) ⇒ No reflection — Black body



Volt	Colour
20 V	No Color — But still Emitting
40 V	Red Color
60 V	orange
80 V	Yellow
100 V	White

Weins displacement

$\boxed{\lambda_m \cdot T = 2.898\times10^{-3}\ mK}$



$\lambda_{20} > \lambda_{40}$

∴ λ_m = Wavelength at which emission is maximum for any particular T.

$\lambda_m \cdot T$ = Constant

2.898×10^{-3} mK

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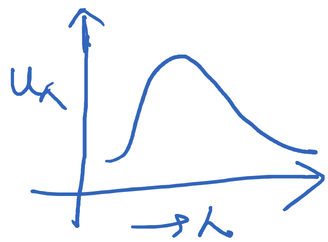
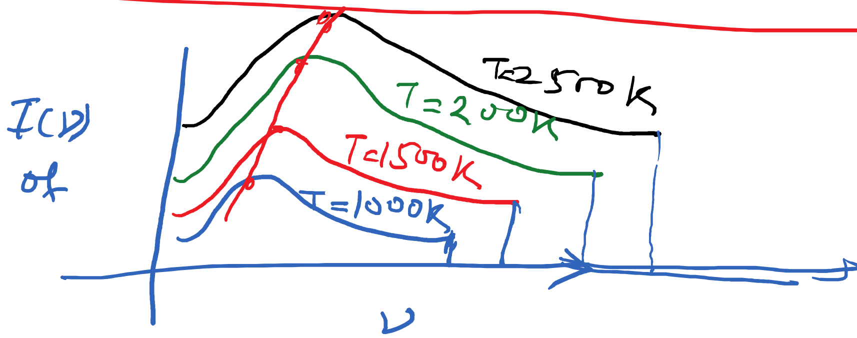
Energy density: $\boxed{U_\lambda}$ → Energy density

Energy of radiation per unit volume in a range from $\boxed{\lambda \text{ to } \lambda + d\lambda}$



$ a good absorber is also a good Emitter.

$ distribution of frequencies is a function of temperature of black body.



Total Radiation

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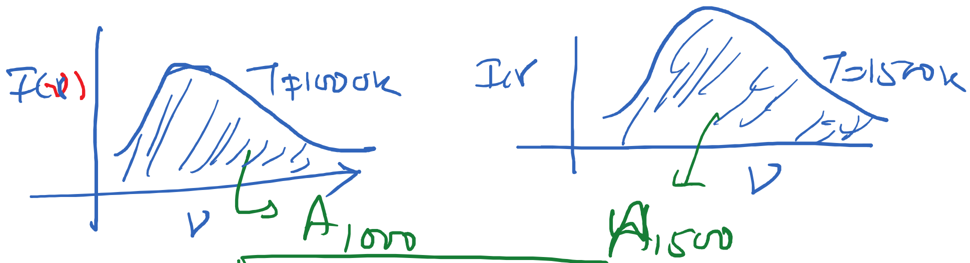


$\Rightarrow$ $\boxed{A_{1500} > A_{1000}}$

- With the increase in temperature the total amount of radiation increases.
- Position of maximum peak shift towards higher frequencies with increasing Equilibrium T.

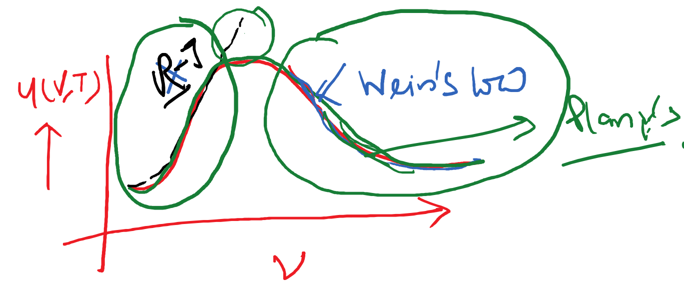
Stefan-Boltzmann

$$\boxed{E = \sigma T^4}$$

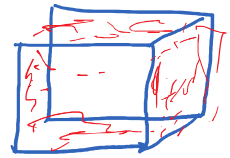
Weins Radiation law

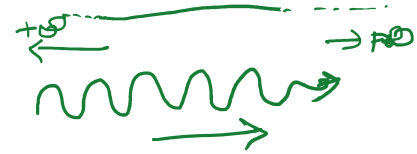
$$\boxed{U(\nu,T) = A\nu^3 e^{-B\nu/T}}$$

No of



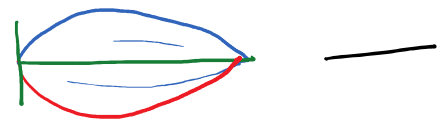
Rayleigh-Jeans law



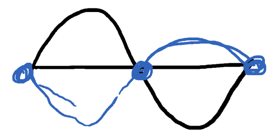


No. of oscillations per unit volume in the frequency range ν to $\nu + d\nu$

$$= \left(\frac{8\pi\nu^2}{c^3}\right)$$



Average energy of oscillators – $\langle E \rangle$



$$\boxed{\langle E \rangle = \frac{\int_0^\infty E e^{-E/kT} dE}{\int_0^\infty e^{-E/kT} dE} = kT}$$

$$\boxed{U(\nu,T) = \frac{8\pi\nu^2}{c^3} kT}$$

Rayleigh-Jeans law

$$\boxed{U(\nu,T) \propto \nu^2}$$

Ultraviolet Catastrophe

Max Planck's Idea:- An oscillator can not have any arbitrary value of energy, but can have only discrete energies as per this formula

$$\boxed{E = n h\nu}$$

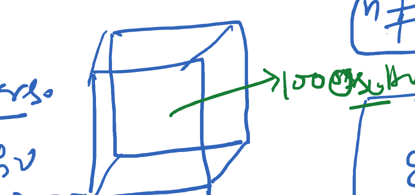
$n \to$ Integer

$n = 0, 1, 2, 3, 4, 5 \ldots$

$\boxed{\cancel{n \neq 1.5, 2.5, 2.2}}$

There are N number of oscillators.

$0,\ h\nu,\ 2h\nu,\ 3h\nu,\ 4h\nu \ldots$



Blue	30
Green	20
Red	40
Yellow	10

$\Rightarrow 100$

$0 - N_0$
$h\nu - N_1$
$2h\nu - N_2$

$N = N_0 + N_1 + N_2 + N_3 \cdots + N_n$

$N_n = N_0 e^{-nh\nu/kT}$

$0 - 10\ N_0$

$h\nu - N_1$
$2h\nu - N_2$
$3h\nu - N_3$
$\vdots$
$nh\nu - N_n$

$$\boxed{N_n = N_0 e^{-nh\nu/kT}}$$

$e^{-E/kT}$

$0 - 10 \; N_0$
$h\nu - 20 - N_1$
$2h\nu - 30 \to N_2$
$3h\nu - 40 \to N_3$

$\to N_0 + N_1 + N_2 + N_3 + \ldots = N$

$$N = N_0 e^{0/kT} + N_0 e^{-h\nu/kT} + N_0 e^{-2h\nu/kT} + N_0 e^{-3h\nu/kT}$$

$$\Rightarrow \boxed{N = \frac{N_0}{1 - e^{-h\nu/kT}}} \quad (1)$$

Total Energy:

$$E = (N_0 \times 0) + (N_1 \times h\nu) + (N_2 \times 2h\nu) + (N_3 \times 3h\nu) + \ldots$$

$$E = N_0 e^{-h\nu/kT} \times h\nu + N_0 e^{-2h\nu/kT} \times 2h\nu + N_0 e^{-3h\nu/kT} \times 3h\nu + \ldots$$

$$\boxed{E = N_0 e^{-h\nu/kT} \frac{h\nu}{(1 - e^{-h\nu/kT})^2}} \quad (2)$$

Total No. of Oscillators $= N$
Total Energy ——— $= E$

$$\boxed{\bar{E} = \frac{E}{N}}$$

$$\boxed{\bar{E} = \frac{h\nu}{e^{h\nu/kT} - 1}}$$

Planck's

$$\boxed{\bar{E} = kT}$$

R-J

$$U_\nu d\nu = \frac{8\pi\nu^2}{c^3} d\nu \times \bar{E}$$

$$\boxed{U_\nu d\nu = \frac{8\pi h}{c^3} \frac{\nu^3 d\nu}{e^{h\nu/kT} - 1}}$$

Planck's Radiation law

Number, Value

$100 \times 2 = 200$

$10 \times 10 = 100$

$10 \times 10 = 100$

Infinity

⇒ Home Work : Deriv Wein's and R-J law Using high and low freq.

Photo Electric Effect :—

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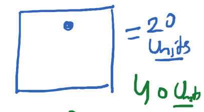
Condition for e^- to come out from the metal

⇒ Minimum Energy to remove the e^- from metal = Work function

$$E_K = h\nu - \phi_0 \quad \text{——— (1)}$$

$$\phi_0 = h\nu_0$$

$$\boxed{E_K = h\nu - h\nu_0} \quad \text{——— (2)}$$



$E = h\nu$

Work function $= \phi_0$

(i) Case i $\quad \nu < \nu_0 \Rightarrow E_K = \underline{\text{Negative}}$ ↳ e^- will not come out from the metal.

(ii) Case ii $\quad \nu = \nu_0 \Rightarrow E_K = 0$ ↳ e^- will come out with no Kinetic Energy.

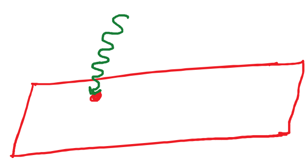
(iii) Case iii $\quad \nu > \nu_0 \Rightarrow E_K = $ Positive ↳ e^- will come out with some kinetic Energy.

ϕ_0 = Work function

⇒ $\boxed{\nu_0 = \text{thershold fresuncy.}}$

$ Calessical View : Light is wave.

Light falls on the metal at $t = 0$ Seconds while e^- Emits at $t = 20$ Secon



$\phi_0 = 20$ Units

$\phi_{wave} = 1$ Unit/sec

Wave will take 20 seconds to transfer 20 Units to e^-.

There may be time lag — b/w Ejection of e^- and falling of light on the metal/surfaes

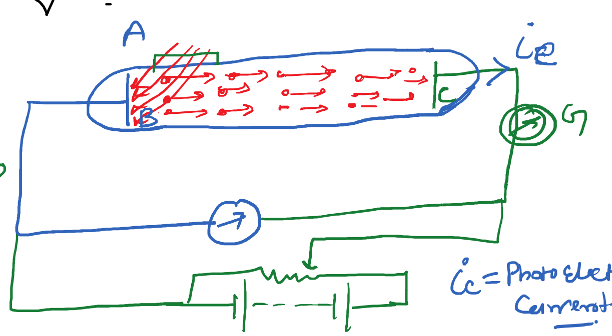
There must be the ejection of e^- irrespective to f or I of the light

There will be no kinetic energy of e^-s.

⇒ Experiment

Plate C is at positive poential with respect to the plate B

(i) Saturation Current.



i_c = Photo Electric Current

(i) +20V — 40 e^- to C

(ii) +40V — 60 e^- to C

+100V — 100 e^- to C

+120V —

Now, the plate C is at negative potential.

Applied potential for which the current $i_e = 0$, is called stopping Potential.

current $I_e = 0$, is called stopping Potential.

$$\boxed{\frac{1}{2} m V_{max}^2 = e|V_0|}$$

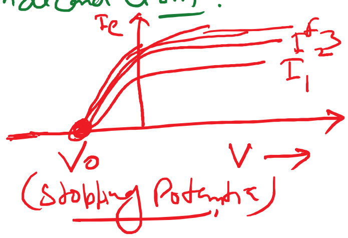
$\hookrightarrow \phi / 20V$

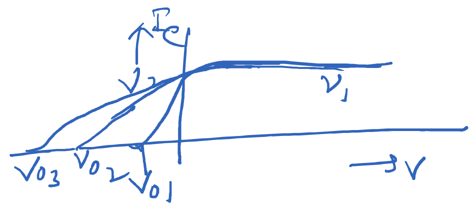
$ Photo Electric current increases with the increasing Intensity of incident radiation, if frequency is kept constant.

$ There is no time lag b/w illumination of the metal surface and the emission of e^-.

$ If the frequency of incident radiation is greater than the threshold frequency, only then emission of e^- takes place.

$ Maximum kinetic energy of photoelectrons is independent of the intensity of the incident light.





$ Maximum kinetic energy of photo electrons depends on the frequency

$E_K = h\nu - h\nu_0$

$ Linear relationship b/w maximum kinetic energy & frequency.

$\Rightarrow$ 1905 Einstein - Solved Photoelectric Effect

Matter Waves :-

Heisenberg's Uncertainty Principle.

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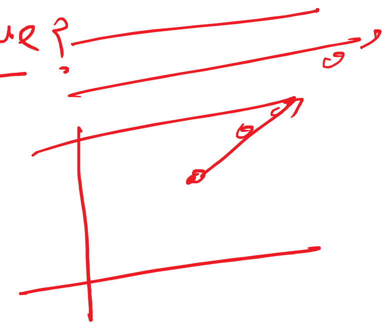
x component of the momentum of particle is measured with an uncertainty Δp_x then its x-position cannot, at the same time, be measured more accurately than $\boxed{\Delta x = \frac{\hbar}{(2\Delta p_x)}}$

$\Delta x \Delta p_x \geq \frac{\hbar}{2}$, $\Delta y \Delta p_y \geq \frac{\hbar}{2}$ and $\Delta z \Delta p_z \geq \frac{\hbar}{2}$ —— (1)

$\boxed{\cancel{\Delta x \Delta p_y \geq \frac{\hbar}{2}}}$ Is it true?

No.



$\Delta x \Delta p_x \geq \frac{\hbar}{2}$

$\Delta y \Delta p_y \geq \frac{\hbar}{2}$

$\cancel{\Delta x \Delta p_y \geq \frac{\hbar}{2}}$

$\Rightarrow \boxed{\frac{\Delta p}{\Delta t} = \Delta F}$ —— (2)

$\Rightarrow \Delta p = \Delta F \cdot \Delta t$

$\Delta x \Delta p \geq \hbar$

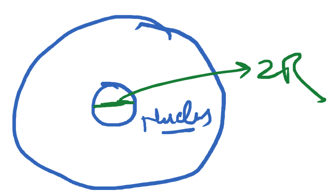
$\Delta x \times (\Delta F \times \Delta t) \geq \hbar$

$[\underline{\Delta x \times \Delta F}]\, \Delta t \geq \hbar$

$\boxed{\Delta E \Delta t \geq \hbar}$

§ Non-Existence of e^- in the Nucleus:

Nucleus $\approx 10^{-14}$ m



Maximum Uncertainity in position of e^-

$(\Delta x)_{maximum} = \underline{2 \times 10^{-14} \text{ meter}}$

$\Delta x \Delta p = \frac{h}{}$ $\Gamma(\Delta x)_{maximum}$

$$\Delta x \Delta p = \frac{h}{2\pi}$$

(Δx) maximum

$\rightarrow (\Delta p)$ minimum

$(\Delta p)_{minimum} \approx$ atleast the e^- willbe having that much momentum.

$$\Delta p = \frac{h}{2\pi \Delta x}$$

$$= \frac{6.625 \times 10^{-34}}{2 \times 3.14 \times 2 \times 10^{-14}}$$

$$\boxed{\Delta p = 5.275 \times 10^{-21} \text{ kg m/sec}}$$

p will not be less than Δp

$$\frac{p^2}{2m} = \frac{1}{2} m v^2$$

$$K.E. = \frac{(5.275 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31}} = 96 \text{ MeV}$$

$\Rightarrow$ 4 MeV

Wave function:—

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$\underline{\psi(r,t)} \longrightarrow$ Wave functions

↓

De Broglie wave of particle ⟶ It describe the wave properties of particle.

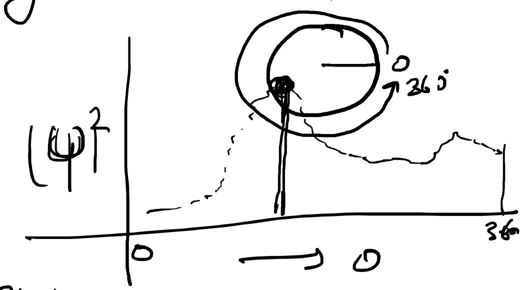
$$\text{Amplitude function} = |\psi|^2 = \psi\psi^*$$

⟶ Complex Conjugate of ψ.

⟶ Equal to the intensity of the wave associated with this Quantum Effect.

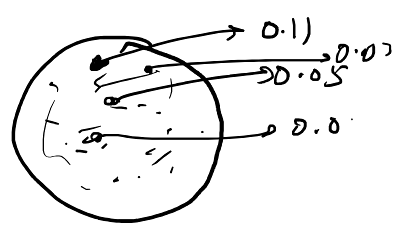
The intensity of a wave at a given point in space is proportional to the ~~finding~~ probability of finding the material particle that corresponds to wave

$|\psi|^2 =$ Probality density.



$|\psi(r,t)|^2 d^3 r$ as the probality, $dP(r,t)$, of finding a particle at time t in a volume element $d3r$ located $\vec{r}$ and $\vec{r}+d\vec{r}$

$$\boxed{\int_{\text{allspace}} |\psi(r,t)|^2 d^3 r = 1}$$



Schrodinger Equation

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$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{u^2}\frac{\partial^2 u}{\partial t^2}$$

$u \rightarrow$ Vertical displacement
$u \rightarrow$ Velocity of wave

$$\hookrightarrow \boxed{\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 u}{\partial t^2}} \rightarrow 3D \quad \text{—— (1)}$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2 \psi}{\partial t^2}} \quad \text{—— (2)}$$

Solution — $\psi(x,y,z,t) = \psi_0(x,y,z)\, e^{-i\omega t}$

$\psi_0(x,y,z)$ is the amplitude of particle at a point (x,y,z) which is independent of time.

$$\Rightarrow \boxed{r = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$\Rightarrow \boxed{\psi(r,t) = \psi_0(r)\, e^{-i\omega t}} \quad \text{—— (3)}$$

$i^2 = -1$

$$\frac{\partial \psi(r,t)}{\partial t} = -i\omega\, \psi_0(r)\, e^{-i\omega t}$$

$$\frac{\partial^2 \psi(r,t)}{\partial t^2} = (-i\omega)(-i\omega)\,\psi_0(r)\, e^{-i\omega t}$$

$$= +i^2\omega^2 \psi_0(r) e^{-i\omega t}$$

$$= -\omega^2 \psi_0(r) e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_0 e^{-i\omega t}$$

$$\boxed{\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi}$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{-\omega^2}{u^2}\psi$$

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{\omega^2}{u^2}\psi = 0 \quad \text{—— (4)}$$

$\omega = 2\pi\nu$

$\omega = 2\pi\left(\frac{u}{\cancel{\lambda}}\right)$

ν (frequency) into wavelength

~~$\lambda = \frac{c}{\nu} = \frac{u}{\nu}$~~

$$\boxed{\nu = \frac{c}{\lambda} = \frac{u}{\lambda}}$$

$$\omega = 2\pi\left(\frac{u}{\cancel{\lambda}}\right)$$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda} \quad — (5)$$

$$\boxed{\nu = \frac{c}{\lambda} = \frac{u}{\lambda}}$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\psi$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad \rightarrow \nabla^2\psi \quad — (6)$$

Putting Equation No. (5) and (6) into Equation no. 4

$$\nabla^2\psi + \left(\frac{2\pi}{\lambda}\right)^2\psi = 0 \Rightarrow \boxed{\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0} \quad — (7)$$

$$\lambda = \frac{h}{p} = \frac{h}{mu}$$

$$\nabla^2\psi + \frac{4\pi^2}{\left(\frac{h}{mu}\right)^2}\psi = 0$$

$$\boxed{\nabla^2\psi + \frac{4\pi^2 m^2u^2}{h^2}\psi = 0} \quad — (8)$$

Total E = K.E. + P.E.

$$E = \frac{1}{2}mu^2 + V$$

$$2m \times \left(\frac{1}{\cancel{2}}mu^2\right) = (E - V) \times 2m$$

$$\boxed{m^2u^2 = 2m(E-V)}$$

Put m^2u^2 in Equation 8

$$\nabla^2\psi + \left(\frac{4\pi^2\, 2m(E-V)}{h^2}\right)\psi = 0 \qquad \left[\hbar = \frac{h}{2\pi}\right.$$

$$\boxed{\nabla^2\psi + \frac{2m(E-V)}{\hbar^2}\psi = 0} \quad — (9)$$

$$\boxed{\nabla^2\psi + \frac{2m(E-V)}{\hbar^2}\psi = 0} \quad \text{— (9)}$$

Time independent Schrodinger Equation.

free particle $\quad V = 0 \quad \boxed{\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0} \quad \text{— (10)}$

Time dependent Schrodinger Equation :-

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$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{u^2}\frac{\partial^2\psi}{\partial t^2} \quad \text{—— (1)}$$

$$\psi = \psi_0(r)\, e^{-i\omega t} \quad \text{—— (2)}$$

$$\Rightarrow \frac{\partial\psi}{\partial t} = -i\omega\, \psi_0 e^{-i\omega t} \rightarrow \psi(r,t)$$

$$\omega = 2\pi\nu$$

Plancks principle

$$\Rightarrow \underline{E = h\nu}$$

$$\nu = \frac{E}{h}$$

$$\frac{\partial\psi}{\partial t} = -i(2\pi\nu)\psi(r,t)$$

$$= -2\pi\nu i\psi$$

$$= -2\pi\left(\frac{E}{h}\right) i\psi$$

$$= -\frac{iE\times i}{\hbar\times i}\psi = \frac{-i^2E}{i\hbar}\psi$$

$$\frac{\partial\psi}{\partial t} = \frac{E}{i\hbar}\psi \quad \Rightarrow \quad \boxed{E\psi = i\hbar\frac{\partial\psi}{\partial t}} \quad \text{—— (3)}$$

$$\underline{(E-V)\psi = E\psi - V\psi}$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}[E\psi - V\psi] = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right] = 0$$

$$\nabla^2\psi = -\frac{2m}{\hbar^2}\left[i\hbar\frac{\partial\psi}{\partial t} - V\psi\right]$$

$$\Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi = i\hbar\frac{\partial\psi}{\partial t} - V\psi$$

$$\Rightarrow -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$$\Rightarrow \boxed{\left(-\frac{\hbar^2}{2m} + V\right)\psi = i\hbar\frac{\partial\psi}{\partial t}} \quad \text{—— (5)}$$

function

operator

Time-dependent Equation

Time-dependent Equation

$\Rightarrow \left(-\frac{\hbar^2}{2m}+V\right)$ is known as Hamiltonian Operator

$$E\psi = i\hbar\frac{\partial\psi}{\partial t} \qquad (3)$$

Compare Eq (3) and (5)

RHS of Eq. 3 = RHS of 5
LHS of Eq 3 = LHS of 5

$$\left(-\frac{\hbar}{2m}\nabla^2+V\right)\psi = E\psi \qquad \Rightarrow E = \left(-\frac{\hbar^2}{2m}\nabla^2+V\right)$$

$$f(x,y,z) = x^3+3y+4z^2$$

$$\frac{\partial f}{\partial x}+\frac{\partial f}{\partial y}+\frac{\partial f}{\partial z}$$